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# ABSTRACT

Using a boundary value treatment, a method is established which gives the internal fields of two coupled radial waveguides. The coupling is achieved by an array of annular slots on the common boundary. It is shown that, in general, higher order modes have significant effect on the solution and for a precise evaluation of the field their contribution must also be included. The possible application of the method for the design of microwave filters with band-pass characteristics is also discussed.

# INTRODUCTION

Two dimensional arrangements of slots or conducting plates to form resonant arrays are of practical interest for applications as band-pass or band-stop filters. Within a certain frequency band, the transmission coefficient of the array can vary from unity to zero and its resonance frequency and bandwidth may be controlled by varying the characteristic dimensions of the array.<sup>1-3</sup>

Different geometries like rectangular, circular or cross type apertures have been investigated as the basic element for array formation. However, little information is available for the case where coupling apertures are of annular shape.

In the present work, the problem of coupling between two radial waveguides by an array of concentric annular slots on the common boundary is considered, figure 1. The slots are of finite width  $\delta_m$  and are located at radii  $\rho = \rho_m$ . The exciting source is placed at the central region of the lower guide and its field can be of a general nature. The slots are assumed to be electrically thin to suppress the  $\phi$  directed electric field component over the apertures.

Using a boundary value treatment, a method of solution is established to derive field expressions for both regions. The formulation includes the presence of two dielectric substrate (with or without loss) to support the structure. We develop our solution by first constructing the appropriate Green's functions describing the impulse response of the system to a magnetic current ring (annular slit) of strength  $I_m$  located at  $\rho = \rho_s$ . Then the final formulation of the problem is achieved by integrating the impulse response over the aperture source distribution.

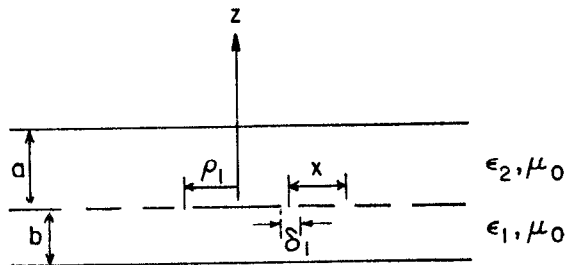


Figure 1: Typical two coupled radial waveguides of infinite extent.

The induced surface magnetic current over the coupling apertures can be expressed by a finite series of suitable basis functions with unknown coefficients.

These constants are then obtained by applying the continuity condition of tangential magnetic field over the apertures. Depending on the required degree of accuracy of the solution and the electrical width of each slot, the aperture field can be represented by a variety of expressions. For electrically narrow slots, field distribution over each aperture may be assumed to be constant with respect to  $\rho$ . In fact, Wait and Hill<sup>4,5</sup> by considering the problem of TEM wave coupling by a circumferential slot on a coated coaxial cable, have shown that differences between the results of higher order approximations for the aperture field and that of the constant field model are inconsequential. Thus, using the constant field approximation, each element of the array is characterized by a complex constant which is yet to be determined by an application of the boundary conditions.

For a waveguide fed array in which apertures are fed successively by a traveling wave, the effect of the mutual coupling between the slots cannot be generally ignored. In the methods based on the waveguide transmission line concept, the coupling between the array elements is usually accounted for only the dominant propagating mode. Furthermore, the coupling due to the external field is generally overlooked. A distinct advantage of the present approach is the fact that all mutual coupling effects and the higher order modes are incorporated automatically in the solution. This is achieved by the simultaneous evaluation of the field unknown coefficients through the application of the boundary conditions.

# Numerical Results and Discussions

Based on the theory presented in the above section, the following cases are considered to illustrate the general feature of the problem. As a check on the accuracy of the numerical results, the balance of the power flow from the incident wave into the transmitted and the coupled waves is examined and for all cases satisfactory results are obtained. Figure 2 depicts the variation of the equivalent admittance of a single annular slot as a function of the radius of its leading edge. The ratio of the radii of the slot edges is kept constant and equal to 2.36. A  $TM_{00}$  excitation is assumed.<sup>6</sup> To obtain an equivalent circuit representation of the slot in terms of an admittance number, we define the admittance  $Y$  of the slot as the ratio of its equivalent current  $I$  and voltage  $V$  in the form

$$Y = \frac{I}{V}$$

$$I = 2\pi H(\rho)$$

$$V = \int_{\rho-\delta/2}^{\rho+\delta/2} E_{\rho}(\rho') d\rho'$$

where  $\rho$  is the radius of the center of the slot. The dotted lines in figure 2 represent the results obtained by taking only the propagating mode, whereas the

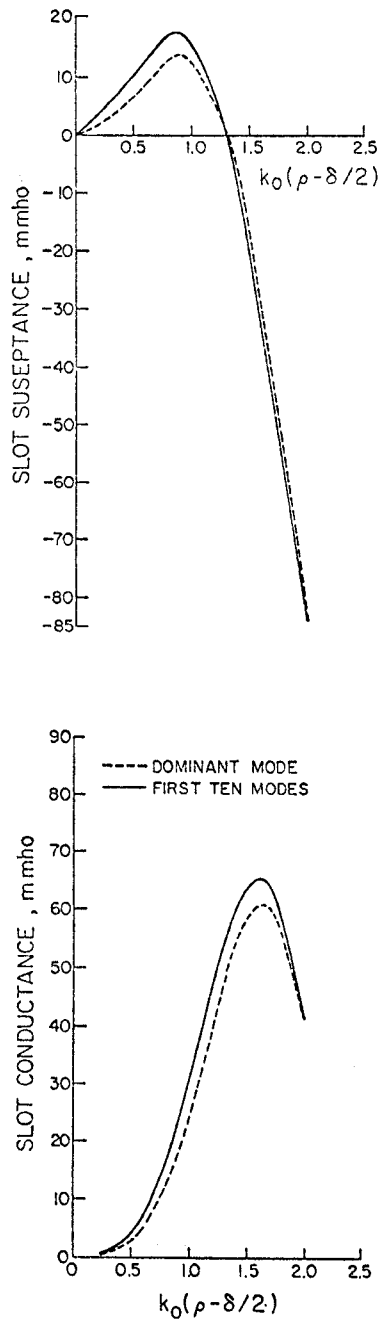


Figure 2: The effect of higher order modes on the equivalent admittance of the slot. Ratio of the edges = 2.36,  $k_0 a = k_0 b = 0.49$ ,  $\epsilon_{r1} = \epsilon_{r2} = 1.00$ .  $k_0$  is free space propagation constant.

solid lines are due to the inclusion of the first ten modes which was observed to be sufficient to yield a convergent solution for all the examples presented in this work. These results indicate that for most applications the dominant mode representation, such as that in the transmission line theory, adequately describes the slot admittance. However, the accuracy of the results, especially for central slots where  $k_0$  is small, is not satisfactory. As a further check on the dominant mode approximation, the field distribution over the slot is also calculated and is shown in figure 3. The slot is located at a radial distance,  $k(\rho - \delta/2) = 2.00$ , which corresponds to the last plotted point of figure 2. Comparing the results one notes that the dominant mode theory yields reasonable results

for the magnitude, except in the vicinity of the slot's leading edge. On the other hand, it fails to describe the phase of the field accurately. The actual phase of the slot field which is calculated by retaining first ten modes is constant across the slot, whereas the result due to the dominant mode oscillates between 0 and  $\pi$  for adjacent matching points. For these calculations, the slot is subdivided into 13 cells (annular rings) of equal widths and the field over each cell is computed by applying the continuity of the tangential magnetic field at its center. These calculations also show that the field distribution of an annular slot is similar to the current distribution on a conducting strip illuminated by a plane wave, Shafai<sup>7</sup>. It should also be noted that even though the dominant mode gives inaccurate phase distribution, it yields reasonable results for the equivalent slot voltage defined earlier. This is due to the fact that the equivalent slot voltage depends on the integral of the slot field and the phase oscillation compensates for the inaccuracies in the slot field amplitude.

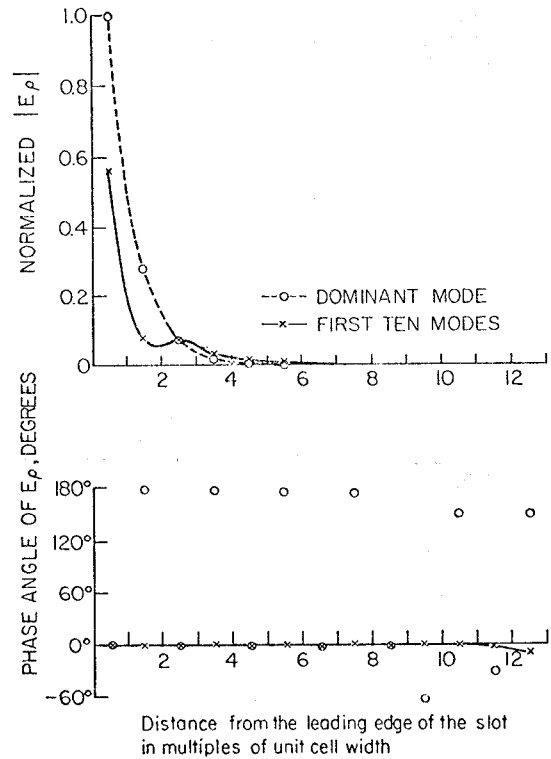


Figure 3: Field distribution over the slot,  $k_0 \delta = 2.72$ ,  $k_0 \rho = 3.36$ ,  $k_0 a = k_0 b = 0.49$ ,  $\epsilon_{r1} = \epsilon_{r2} = 1.00$ .

Figures (4-6) show the application of annular slot array as band-pass filters. Since the slots are electrically narrow, the calculations are carried out by representing each slot by a single cell. That is, each slot is characterized by a constant complex number. The filtering characteristics of a single slot excited by a  $TM_{00}$  radial mode is shown in figure 4. It is interesting to note that by a proper selection of the geometry, a single narrow slot can effectively couple a major part of the total power to the upper guide over a wide frequency band with a sharp cut-off characteristic.

Figure 5 illustrates the transfer function of a filter having four slots as its coupling elements for the  $TM_{00}$  excitation. The response of the same geometry to the  $TM_{01}$  exciting mode is shown in figure 6. The resonance peak is slightly shifted toward higher frequencies and the bandwidth is decreased compared to  $TM_{00}$  mode of operation. This figure also shows the

sensitivity of the filter characteristic to the slot spacing. The dotted curve is obtained by increasing the spacing by an amount equal to four per-cent.

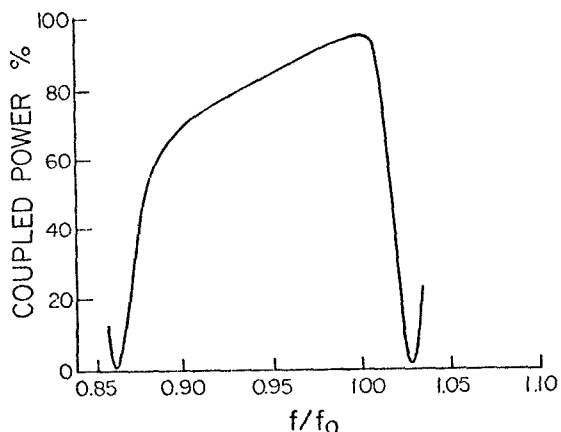


Figure 4: Filter characteristic of a single slot,  $TM_{00}$  exciting mode.  $\epsilon_{r1}=2.60$ ,  $\epsilon_{r2}=5.20$ ,  $k_0a=0.34$ ,  $k_0b=0.04$ ,  $k_0\delta=0.05$ ,  $k_0\rho=8.39$ . All dimensions are at  $f_0$ .

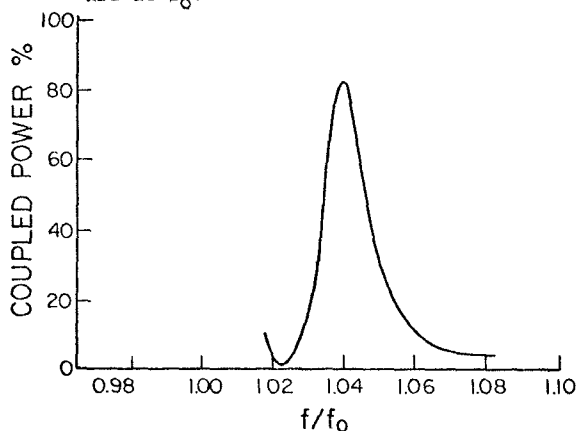


Figure 5: Filter characteristic of four slots  $TM_{00}$  exciting mode.  $\epsilon_{r1}=2.60$ ,  $\epsilon_{r2}=5.20$ ,  $k_0a=k_0b=0.34$ ,  $k_0\delta=0.08$ ,  $k_0\rho_1=10.08$ ,  $k_0x=4.05$ .

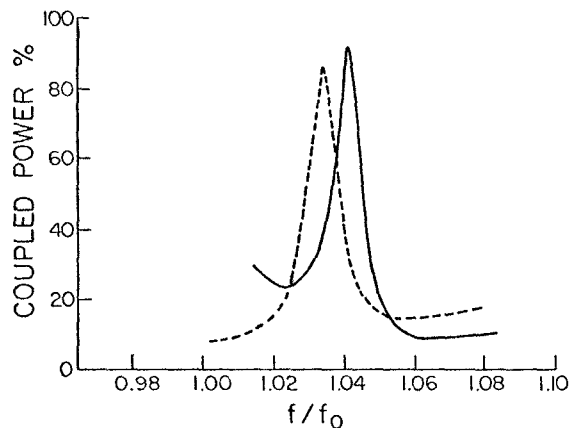


Figure 6: Filtering response of the same geometry as in figure 5 to  $TM_{01}$  mode of operation. The dotted curve is due to a 4% increase in slot spacing  $k_0x$ .

The inherent high frequency capability of waveguide structures and the simplicity of the present model is promising for use in microwave stripline circuits as a filter or coupler for dual feed systems. A thorough study of the slot admittance and its relation to the filtering characteristics is presently

under investigation.

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